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obliquities of the two surfaces (*i. e.* the angles which the elements make with their respective axes); then the required condition is

$$r_1 : r_2 = \tan i_1 : \tan i_2.$$

Take, as in the Fig., the plane through the axis of the first surface, parallel to the axis of the second as the vertical, and a plane perpendicular to the axis of the first surface, as the horizontal plane. Since the surfaces are tangent, and the plane tangent to both at the point where their gorge circles touch each other contains the common element and is parallel to the vertical plane, the common element is parallel to the vertical plane and its vertical projection is a common asymptote to the vertical projections of the surfaces.

At any point of the element of contact, as at N, (n, n') draw a common normal to the two surfaces- It intersects both axes, one in O, and the other in P.

Since $M\ N$ is parallel to vertical plane, $o'\ n'\ p'$ must be perpendicular to $m'\ n'$. Now the horizontal projecting plane of $M\ N$ divids the perpendicular between the axes into r_1 and r_2, and the normal into $O\ N$ and $N\ P$; hence

$$r_1 : r_2 = O\,N : N\,P$$
$$= o'\,n' : n'\,p'.$$

The obliquities of the surfaces are given in full size in the vertical projection; that is $i_1 = o'\,m'\,n'$ and $i_2 = n'\,m'\,p'$.

Hence, $\tan i_1 : \tan i_2 = o'\,n' : n'\,p'$

and $r_1 : r_2 = \tan i_1 : \tan i_2.$

Q. E. D.

SOLUTIONS OF PROBLEMS IN NOS. 3 AND 4.

Solutions of problems in No. 4, have been received as follows: From Prof. W. E. Arnold, 16 and 18; R. J. Adcock, 17; Marcus Baker, 16, 17, 18 & 19; S. J. Child, 16 & 18; George L. Dake, 16; Prof. A. B. Evans, 16, 17, 18 & 19; Henry Gunder, 16, 17, & 18; Prof. E. W. Hyde, 16, 17, & 18; William Hoover, 16; Phillip Hoglan, 18; Prof. I. N. Jones, 16; J. B. Mott, 16; L. E. Newcomb, 16; A. W. Mason, 16; Prof. A. W. Phillips, 16, 17, 18 & 19; L. Regan, 16; John W. P. Reid, 16; Henry A. Roland, 17 & 19; Miss Kittie Robinson, 16; Prof. Selden Sturges, 16, 17 & 18; E. B. Seitz, 16, 17, 18 & 19; S. W. Salmon, 16, 17, 18 & 19;

R. L. Selden, 16; Walter Siverly, 16, 17, & 19; Elias Schneider, 16; Jas. Stott, 16 & 18; Prof. D. M. Sensenig, 16, 17, 18 & 19; Prof. M. C. Stevens, 16, 17 & 18; and Prof. David Trowbridge, 16, 17, 18 & 19.

14. "Two equal particles attracting each other with forces varying inversely as the square of the distance, are constrained to move in two straight lines at right angles to each other; supposing their motions to commence from rest, to find the time in which each of them will arrive at the intersection of the two straight lines."

SOLUTION BY PROF. J. M. GREENWOOD, KIRKSVILLE, MO.

Take the lines as coördinates, the origin being at their intersection; let a and b represent the distances of the particles from the origin, and x and y their distances at the time t.

From the problem (1) $a^2 + b^2 = c^2$, (2) $x^2 + y^2 = r^2$.

The equations of motion are

$$(3)\quad \frac{d^2 x}{d t^2} = -\frac{\mu x}{r^3}, \quad (4)\quad \frac{d^2 y}{d t^2} = -\frac{\mu y}{r^3}.$$

$$(5)\quad \frac{y\, d^2 x}{d t^2} - \frac{x\, d^2 y}{d t^2} = 0 \text{ ; also } (6)\quad \frac{y\, d x}{d t} = \frac{x\, d y}{d t}.$$

$$(7)\quad \frac{d x}{x} = \frac{d y}{y}; \therefore \log \frac{x}{a} = \log \frac{y}{b}.$$

$$(8)\quad \therefore \frac{x}{a} = \frac{y}{b} = \frac{r}{c}.$$

When $x = 0$, $y = 0$, $r = 0$, $\because$ the particles arrive at the origin simultaneously.

To find the time when the force varies inversely as the square of the distance we have (Smith's Mechanics, page 207).

$$(9)\quad d t = \frac{-d r}{\sqrt{2 \mu}\sqrt{\left(\frac{c-r}{c r}\right)}} \qquad \text{Intregating (9)}$$

$$(10)\quad t = \left(\frac{c}{2 \mu}\right)^{\frac{1}{2}} \left\{ (c r - r^2)^{\frac{1}{2}} - \frac{c}{2} \text{ versin.}^{-1} \frac{2 r}{c} + C \right\}.$$

$$\text{When } t = 0,\ r = c;\ \therefore C = \frac{r \pi}{2} = \frac{c \pi}{2}.$$

$$(11)\quad t = \left(\frac{c}{2\mu}\right)^{\frac{1}{2}} \left\{ (c\,r - r^2)^{\frac{1}{2}} + \frac{c}{2}\ \left(\pi - \text{versin}^{-1}\frac{2\,r}{c}\right), \right\}.$$

$$\text{when } r = 0,\ t^2 = \frac{\pi^2\,c^2}{8\,\mu}\ ;\ \therefore t = \frac{\pi\,c^{\frac{3}{2}}}{(8\mu)^{\frac{1}{2}}}.$$

* * *

15. "Two points are taken at random in the surface of a given circle and a chord drawn through each at random; show that the chance of their intersecting is $\frac{1}{2} + \frac{5}{2\,\pi^2}$."

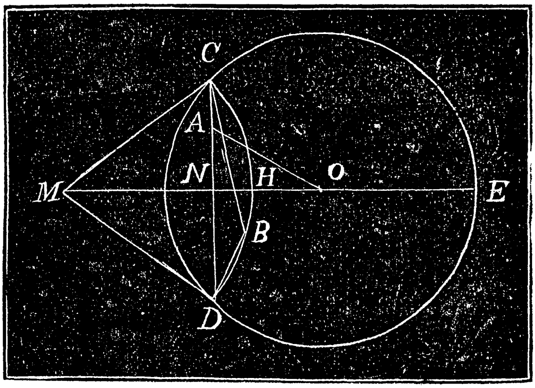
SOLUTION BY E. B. SEITZ, GREENVILLE, OHIO.

Let $C\,E\,D$ be the given circle, O its center, A, B the random points, $C\ D$ the random chord through A, $C\,H\,D$ an arc through B, M its center.

Put $O\,A = x$, $N\,H = y$, $C\,N = m$, $O\,G = 1$, $< O\,A\,D = \theta$, $< C\,M\,N = \phi$, area seg. $C\,H\,D = s$, area sector $C\ M\ D = V$, area $\triangle\ C\,M\,D = t$. Then $O\,N = x \sin\,\theta$, $N\,E = 1 + x \sin\,\theta$, $N\,G = 1 - x \sin\,\theta$, $m = (1 - x^2 \sin^2\,\theta)^{\frac{1}{2}}$, $C\,M = \frac{m^2 + y^2}{2\,y}$, area seg. $C\,E\,D = \pi - \sin^{-1}\,m + m\,x\,\sin\,\theta$, area seg. $C\,G\,D = \sin^{-1} m - m\,x\,\sin\,\theta$, $<$

$$C\,B\,D = \pi - \phi,\ \phi = \sin^{-1}\left(\frac{2\,m\,y}{m^2 + y^2}\right),\ V = \phi\left(\frac{m^2 + y^2}{2\,y}\right)^2,$$

$$t = \frac{m\,(m^2 - y^2)}{2\,y},\ s = V - t.$$



Now when B is in the segment $C\,E\,D$ the limits of y are 0 and $1 + x \sin\,\theta$, of x, 0 and 1, and of θ, 0 and $\frac{1}{2}\,\pi$; and when B is in the segment $C\ G\ D$ the limits of y are 0 and $1 - x \sin\,\theta$, of x, 0 and 1, and of θ, 0 and $\frac{1}{2}\,\pi$. Hence the required chance is

$$p = \frac{\int_0^{\frac{1}{2}\pi}\int_0^1 \left\{ \int_{y=0}^{y=1+x\sin\theta} 2(\pi - \phi)d\,s + \int_{y=0}^{y=1-x\sin\theta} 2(\pi - \phi)d\ s \right\} 2\,\pi\,x\,d\,x\,d\,\theta}{\int_0^{\frac{1}{2}\pi}\int_0^1 \left\{ \int_{y=0}^{y=1+x\sin\theta} 2\,\pi\,d\,s + \int_{y=0}^{y=1-x\sin\theta} 2\,\pi\ d\ s \right\} 2\,\pi\,x\,d\,x\,d\,\theta}$$

$$=\frac{2}{\pi^3}\int_0^{\frac{1}{2}\pi}\int_0^1 \left\{ [\pi s - V\psi]_{y=0}^{y=1+x\sin\vartheta} + [\pi s - V\psi]_{y=0}^{y=1-x\sin\vartheta} \right\} 2\,x\,d\,x\,d\,\theta$$

$$=\frac{4}{\pi^3}\int_0^{\frac{1}{2}\pi}\int_0^1 \left\{ \pi \sin^{-1} m - [\sin^{-1} m]^2 + m^2 \right\} 2\,x\,d\,x\,d\,\theta$$

$$=\frac{1}{\pi^3}\int_0^{\frac{1}{2}\pi} [\pi^2 - 4\,\theta^2 + 2\,\theta^2 \operatorname{cosec}^2\theta - 4\,\theta\cot\theta + 5 + \cos 2\,\theta]\,d\,\theta$$

$$=\frac{1}{\pi^3}\left[\pi^2\,\theta - \frac{4}{3}\theta^3 - 2\,\theta^2\cot\theta + 5\,\theta + \tfrac{1}{2}\sin 2\,\theta\right]_0^{\frac{1}{2}\pi} = \frac{1}{3} + \frac{5}{2\pi^2}.$$

16. "What are the sides of a right angled triangle the area of which is 5 acres, and one of its acute angles 22° 30′?"

SOLUTION BY E. B. SEITZ, GREENVILLE, OHIO.

Let x = the side adjacent to the given angle; then x tan 22° 30′ = the side opposite, and the area of the triangle is

$$\frac{1}{2}x^2 \tan 22^\circ\, 30' = 800; \text{ whence}$$

$$x^2 = 1600 \cot 22^\circ\, 30' = 1600\left(\frac{1+\cos 45^\circ}{\sin 45^\circ}\right) = 1600\,(\sqrt{2}+1);$$

$$\therefore x = 40\sqrt{\sqrt{2}+1}, \text{ and } x \tan 22^\circ\, 30' = 40\sqrt{\sqrt{2}-1}.$$

17. "Find the sum of the series

$$a\,r^n + (a+b)\,r^{n-1} + (a+2\,b)\,r^{n-2} + \ldots (a+n\,b) = S".$$

SOLUTION BY PROF. D. TROWBRIDGE, WATERBURGH, N. Y.

Multiply the given series by r and subtract the given series from the product, then

$$S\,(r-1) = a\,r^{n+1} - n\,b + b\,r\,(r^{n-1} + r^{n-2} + \ldots + 1) - a$$

$$\therefore S = \frac{a\,(r^{n+1}-1)}{r-1} + \frac{b\,r\,(r^n-1)}{(r-1)^2} - \frac{n\,b}{r-1}.$$

If $b = 0$, we have the ordinary geometrical series.

18. "Find the differential coefficient of the function $(1+x^m)^n.(1+x^n)^m$ and express the answer in its simplest form."

SOLUTION BY PROF. A. W. PHILLIPS, CHESHIRE, CT.

$$\text{Put } u = (1 + x^{m})^{n} . (1 + x^{n})^{m}.$$

$$\frac{du}{dx} = n\,m\,x^{m-1}(1 + x^{m})^{n-1}(1 + x^{n})^{m} + n\,m\,x^{n-1}(1 + x^{n})^{m-1}(1 + x^{m})^{n},$$

$$\frac{du}{dx} = n\,m\,u\left\{\frac{x^{m-1}}{(1+x^{m})} + \frac{x^{n-1}}{(1+x)^{n}}\right\}.$$

19. "Referring to question 4, (No. 1): At what time will the lower cask contain the greatest quantity of wine?"

SOLUTION BY M. BAKER, U. S. C. S. SANFRANCISCO, CAL.

From Mr. S. W. Salmon's solution (No. 3), we have

$$Q = c\,t\,e^{-\frac{c\,t}{a}}$$

in which Q equals the amount of wine in the lower cask at the end of t minutes. Now the question is what value of t renders Q a maximum?

Differentiating we have $\frac{d\,Q}{d\,t} = c\,e^{-\frac{c\,t}{a}} - \frac{c^2}{a}\,t\,e^{-\frac{c\,t}{a}}$.

Making this equal zero and solving, $t = \frac{a}{c}$, *i. e.* there will be the greatest amount of wine in the lower cask when all the pipes have been open for the time $\frac{a}{c}$ and the quantity of wine in the lower cask at the end of $\frac{a}{c}$ minutes is $\frac{a}{e}$ ($=.\ 36788a$).

SOLUTION BY PROF. D. M. SENSENIG, MILLERSVILLE, PA.

Let x represent the number of instants in a minute and u the quantity of wine in the lower cask at the end of t minutes, then will

$$u = t\,c\left(1 - \frac{c}{a\,x}\right)^{t\,x-1}, \quad \text{which is a maximum.}$$

$$\frac{d\,u}{d\,t} = \left\{1 + t\,x \log'\left(1 - \frac{c}{a\,x}\right)\right\}\left\{1 - \frac{c}{a\,x}\right\}^{t\,x-1} = 0$$

$$\therefore 1 + t\,x \log'\left(1 - \frac{c}{a\,x}\right) = 0,$$

$$\text{or } 1 - \frac{t\,c}{a} = 0\text{ , since } x = \infty; \quad \text{whence } t = \frac{a}{c}.$$

In solving the above problem, I observed the following beautiful law: Suppese the number of casks containing water to be indefinitely increased, and the other conditions to remain the same. Develop into a series the expression $\left\{\left(1-\frac{c}{ax}\right)+\frac{c}{ax}\right\}^{tx}$, in which x represent the number of instants in a minute, thus:

$$\left\{\left(1-\frac{c}{ax}\right)+\frac{c}{ax}\right\}^{tx}=\left(1-\frac{c}{ax}\right)^{tx}+tx\left(1-\frac{c}{ax}\right)^{tx-1}\frac{c}{ax}$$
$$+\frac{tx(tx-1)}{2}\left(1-\frac{c}{ax}\right)^{tx-2}\frac{c^2}{a^2x^2}+\text{etc.}$$

If we now represent the parts of wine in each cask at the end of t minutes, beginning with the first, respectively by u, u', u'', u''' ets., then will

$u=\left(1-\frac{c}{ax}\right)^{tx}$, a minimum when $t=\infty$

$u'=tx\left(1-\frac{c}{ax}\right)^{tx-1}\frac{c}{ax}$, a maximum when $t=\frac{a}{c}$

$u''=\frac{tx(tx-1)}{2}\left(1-\frac{c}{ax}\right)^{tx-2}\frac{c^2}{a^2x^2}$, a maximum when $t=\frac{2a}{c}$

$u'''=\frac{tx(tx-1)(tx-2)}{2.\ 3}\left(1-\frac{c}{ax}\right)^{tx-3}\frac{c^3}{a^3x^3}$ a maximum when $t=\frac{3a}{c}$

etc., etc., etc.

The values of u, u' u'', u''', etc., may readily be found by developing the above expressions by the binomial theorem and substituting $x=\infty$ or by applying the principles of logarithms as the student may prefer.

PROBLEMS.

25. By R. L. Seldon, Troy, N. Y.—Required the sides of an obtuse angled triangle the area of which is 14.048 acres, the obtuse angle 111°15′, and one of the acute angles 11° 44′ 10″.

26. By Wm. Hoover, South Bend, Ind.—Find θ from the equation $15\sin\theta+12\cos\theta=17.\ 97240$..................................(1).

27. By D. J. McAdam, Washington, Pa.—Four given equal spheres being placed in close contact with each other, it is required to find the volume of the space inclosed between them and the three triangular planes through each three centers.

29. By Prof. A. B. Evans, Lockport N. Y.—If $a, b, c, d, e, f, g, h, i, j, k$ be chords drawn from any point on the circumference of a circle to the eleven angles of an inscribed regular polygon of eleven sides; prove that $(a+k)(b+j)(c+i)(d+h)(e+g)=f^5$..................(1)